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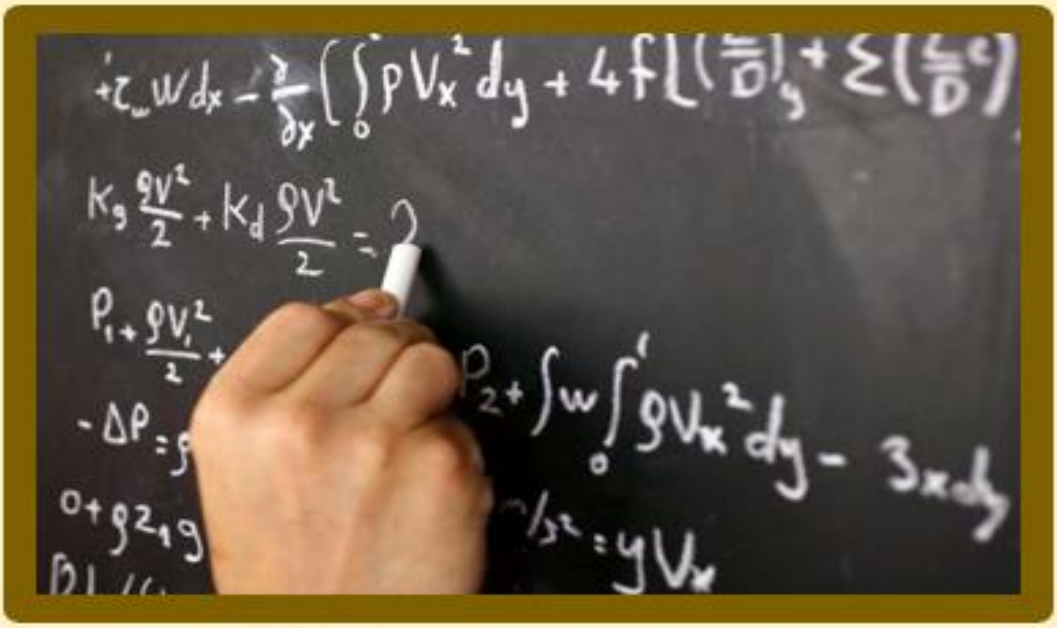

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MATHEMATICAL MODELING OF CULTISM DYNAMICS AMONG YOUTH IN SOUTHERN NIGERIA

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Abstract: *Cultism is believed to be one of the major crimes in Nigeria and this has harmed the reputation of Nigerian institutions, as well as the quality of learning in Nigeria. In consideration of this, a mathematical model of cultism was developed. The model was constructed to analyze and control the spread of cultism activities in southern Nigeria. The model comprises six (6) compartments: Susceptible individuals (S), Aiders (S_c), Cultists (C), Apprehended individuals (A), Escaped individuals (E), and Government efforts (G). The model assesses control strategies by establishing a basic reproduction number which shows the spread of cultism activities. A reproduction number less than one indicates the potential eradication of cultism activities otherwise indicates persistence. The global stability was constructed using Lyapunov theorem. The numerical simulation revealed that rescuing of apprehended individuals from cultists and killing of cultists individuals seems too good for reducing cultism activities in southern Nigeria.*

keywords: Modeling, Cultism, Stability, and Reproduction Number

1. INTRODUCTION

Cultism is described as a secretive ritual practiced by a group of individuals whose membership acceptance, policy and initiation procedures, as well as their manner of operation have negative consequences for both members and non-members, (Ajayi, 2015).The largest and most humiliating challenge confronting Nigeria's higher institutions today is the resurgence of the threat and aggressiveness of cult activities, (Amunnadi, 2017). Cultism is believed to be the mother of all crimes in tertiary institutions and this issue has harmed the reputation of our institution, as well as the quality of learning and the integrity of the grandaunts Salisu, (2021). A secret cult is a closed, structured group of people with a common goal. It is a closed group with philosophy and a set of ceremonies that revolve around its hidden symbols. Blood is occasionally included in cult activities. It might be the blood of an animal or that of a person. when confraternities first appeared in the 1950s, they were not violent. Unfortunately, they were eventually hijacked by military regimens eager to strengthen their grip on university students who may rebel against them. Military officials, for example, saw virulent student unionism as a challenge to their power consolidation. Oguande (2002). As a result, covert cults were employed to suppress student unions and their anti-government activities. In the 1950s and 1960s, student organizations, which initially served as platforms for intellectual and social discourse, began to take on secretive and exclusive characteristics. These groups, such as the Pirates Confraternity were founded by seven students including Wole Soyinka and Ralph Okpara (modeled after the Skull and Bones Society in the United States). The 1970s and 1980s were marked by political unrest and economic challenges in Nigeria. Cults started to proliferate as young people sought a sense of belonging and empowerment outside the deteriorating societal structure while in the 1990s, cultism expanded beyond university campuses and infiltrated Nigerian society at large. Recognizing the grave implications of cultism, the Nigerian government initiated various measures to curb its spread. The introduction of stringent Anti-cultism Laws and the establishment of specialized law enforcement units such as the Anti-cultism Unit of the Nigerian Police Force, aimed to dismantle cults. Five thousand students and professors have died in Nigerian universities as a result of

cult-related violent conflicts. Benson (2002). The Pirates confraternity, also known as the Sea Lord, was founded as a student protest group at the University of Ibadan in 1952. Okwu, (2006). Thomas (2002) investigated the hazards of secret cults in Nigerian tertiary institutions.
Cultism is one of the major social problems like banditry, violence, and terrorism, etc. mathematicians across the world such as (Aniayam et al. (2018), Adamu & Ibrahim, (2020), Okoye et al (2021), Hussaini (2019), and Brauer (2012)) contributed toward curbing the menace through mathematical modeling, for more than a decade now, cultism has been a serious social problem more especially in southern Nigeria. Akanni & Abidemi(2021) developed a mathematical model of the Relationship between Illicit Drug Users and Bandits in a Population. Among the major global health and social problems facing the world today are the use of illicit drugs and the act of banditry. Also, John et al (2023) also established Forces of Terror: Armed Banditry and Insecurity in Northwestern Nigeria. The authors stated that Nigeria had confronted several security conundrums in recent years, including armed banditry, which poses a severe threat to the northwest and the entire nation, Similarly, Lawal et al (2023) constructed a mathematical model and Optimal Control Analysis on Armed Banditry and Internal Security in Zamfara State. Akanni (2020) also modeled the Asymptotic Stability Of Illicit Drug Dynamics with Banditry Compartment. Tasiu et al (2024) developed a mathematical model of kidnapping dynamics and control, the authors categorized the kidnappers by their activities, and their model assessed control strategies such as rehabilitation of and eliminating kidnappers Daniel (2022) established a Mathematical Model of Banditry and Security Threats: An Analysis of insecurity in Nigerian. The author states that Nigeria has remained a violent-ridden nation on the psycho-political map of the world for several decades. To the outside world, poverty, civil disturbances, Cultism, guerilla warfare, insurgence, diseases, domestic rebellion, terrorism, revolt and in recent times banditry are the core features of the continent, especially Nigeria. Hence this work focuses on developing a mathematical model for cultism dynamics in southern Nigeria.

2. MODEL FORMULATION

2.1. Model Description.

In this research, the total population at time, (t) denoted by N(t) is split into the mutually exclusive compartments: Susceptible individual S(t), Aiders $S_c(t)$, Cultist C(t), Apprehended individual A(t), Government efforts G(t), and Escaped individual E(t), each compartment represents a different role in cultism network.The schematic diagram is represented in figure (1) below

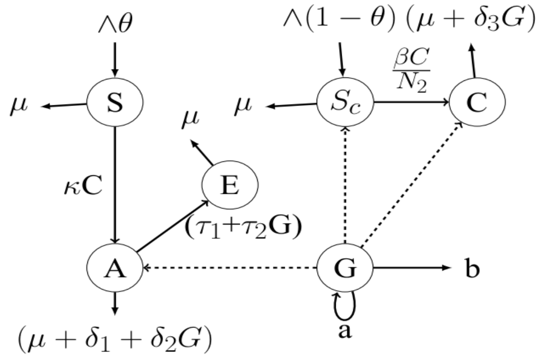

Figure 1: Schematic diagram of cultist model

The Susceptible individualS(t) is generated by daily recruitment by either birth/immigration at the rate $\wedge\theta$, where θ is the proportion of recruitment, and diminished by individuals that are apprehended by cultists at the rate κ,. The Aiders, $S_c(t)$ that is those that provide cultists with information and charm, they are generated by daily recruitment through birth/immigration at the rate $\wedge(1-\theta)$. The population reduces at the rate $\frac{\beta C}{N_2}$. The population of the Cultist individual,$C(t)$ is generated by the progression from Aiders individuals at the rate $\frac{\beta C}{N_2}$ and diminished by the death of cultist by Government δ_3. The population of the Apprehended, $A(t)$ is formed by progression from the susceptible class at the rate κ. This population is diminished by individuals who escaped from cultists, by either escaping or being rescued by Government at the rate of τ_1 and τ_2, This class is also reduced by death due to cultist activity δ_1, and death due to collateral damage δ_2. The population of the Escaped, $E(t)$ is generated by people who either escaped from cultist τ_1 or rescued by government efforts τ_2. We assume natural death μ in all the compartments. We assumed that there is a homogeneous mixing of the population whereby all susceptible can be apprehended.
The corresponding mathematical representation of the schematic diagram in (2.1) is given by equation

(1) below

$$
\left.\begin{aligned}
\frac{dS}{dt} &= \wedge\theta - \kappa CS - \mu S \\
\frac{dS_c}{dt} &= \wedge(1-\theta) - \frac{\beta CS_c}{N_2} - \mu S_c \\
\frac{dC}{dt} &= \frac{\beta CS_c}{N_2} - (\mu + \delta_3 G)C \\
\frac{dA}{dt} &= \kappa CS - (\tau_2 + \delta_2)GA - (\mu + \tau_1 + \delta_1)A \\
\frac{dE}{dt} &= (\tau_1 + \tau_2 G)A - \mu E \\
\frac{dG}{dt} &= a - bG
\end{aligned}\right\} \tag{1}
$$

. and let

$$
\lambda = \frac{\beta C}{N_2}, \tag{2}
$$

where

$$
\left.\begin{aligned}
N_1 &= S + A + E \\
N_2 &= S_c + C \\
N &= N_1 + N_2
\end{aligned}\right\}. \tag{3}
$$

So that

$$
\frac{dN}{dt} = \wedge - \mu N - (\delta_1 A + \delta_2 GA + \delta_3 GC) \tag{4}
$$

.

Table 1.1: Table for variables and parameters of the model

Variables and Parameters	Description
S	Susceptible individual
S_c	Aiders
C	Cultists individuals
A	Apprehended individuals
G	Government effort
E	Escaped individuals
$\wedge$	Recruitment rate
θ	Proportion of recruitable individuals
κ	Rate of apprehension
β	Effective contact rate
a	Government capacity
b	Relaxation of Government
τ_1	Escaped
τ_2	Rescue by Government
δ_1	Death of captured individual by cultist
δ_2	Collateral damage
δ_3	Death of cultist by Government
μ	Natural death
N	Total number of the population

3. Basic Properties of Cultist Model

3.1. Invariant Region:

The population size can be determined by linear differential equation of the model formulated.

$$
\frac{dN}{dt} = \frac{dS}{dt} + \frac{dS_c}{dt} + \frac{dC}{dt} + \frac{dA}{dt} + \frac{dE}{dt} + \frac{dG}{dt}, \tag{5}
$$

i.e

$$
\frac{dN}{dt} = \wedge - \mu N - (\delta_1 A + \delta_2 GA + \delta_3 GC)
$$

such that

$$
\frac{dN}{dt} \le \wedge - \mu N. \tag{6}
$$

then N = S + $\mathbf{S}_c$ + C + A + E + G and equation (5) resolved to linear differential equation of the form;

$$
\frac{dN}{dt} + \mu N \le \wedge. \tag{7}
$$

Theorem 3.1: The solution of the system of non-linear equation (1) is feasible for $t < 0$, if they are in invariant region Ω.
Proof: Let $(S, S_c, C, A, F, G) \in \Re^6$ be any solution of the system with non-negative initial conditions using integrating factor

$$I.F = \exp^{\int pdt} = \exp^{\mu\, t}+C = \exp^{\mu\, t}.\exp^{c} = A\exp^{\mu\, t}$$

$$A\exp^{\mu\, t}\frac{dN}{dt} = A\exp^{\mu\, t}\wedge$$
$$A\exp^{\mu\, t}dN = \wedge A\exp^{\mu\, t}dt.$$

so that

$$N(t) = \frac{\wedge}{\mu} + C\exp^{-\,\mu t}. \tag{8}$$

as t = 0, the initial population will become

$$N(0) = \frac{\wedge}{\mu} + C, \tag{9}$$

where C is constant.
By simplification, we have that

$$C = N_0 - \frac{\wedge}{\mu}and$$

by substitution, We have

$$N(t) = \frac{\wedge}{\mu}(1 - \exp^{-\,\mu t}) + N_0\exp^{-\,\mu t}$$
$$N_0 \le \frac{\wedge}{\mu}.$$

As $t \to \infty$, the human population approaches $C = \frac{\wedge}{\mu}$, where $C = \frac{\wedge}{\mu}$ is the carrying capacity.
Hence, all feasible solution set of the population of the model system in (1) entered the region;

$$\Omega = S, S_c, C, A, E, G \in \Re^6 : S, S_c, C, A, E, G \ge 0$$

$$\therefore N \ge \frac{\wedge}{\mu}. \tag{10}$$

Hence, it's positively invariant set under the flow induced by the model, hence the model is well-posed in the domain.

3.2. Positivity of the Solutions

The following results guarantee by the cultist model governed in the equation (1), is well-posed in a feasible region, Ω.

Theorem 3.2: The solutions of the system in (1) with positive initial condition will remain positive for all time, $t \ge 0$.
proof: Using The equations of the model (1), we have,

$$\frac{dS}{dt} = \wedge\theta - \kappa CS - \mu S.$$

so that

$$\frac{dS}{dt} = \wedge\theta - (\kappa C + \mu)S. \tag{11}$$

By using separation of variables, we obtain

$$\frac{dS}{S} \ge -(\kappa C + \mu)dt \tag{12}$$

and by Integrating both sides of (12) we get,

$$\int \frac{dS}{S} \ge -\int (\kappa C + \mu)dt \tag{13}$$

which implies that

$$\log(S) \ge -(\kappa C + \mu)t + \log c \tag{14}$$

$$S(t) \ge \exp^{-(\kappa C+\mu)t}. \tag{15}$$

as t=0, we have

$$S(0) \ge c\exp^{-(\kappa C+\mu)(0).}$$

and $S(0) \ge c$.
Similarly, it can be shown that ($S_c(t)$, C(t), A(t), E(t), and $G(t) > 0$ for all $t \ge 0$.

3.3. Cultist-free Equilibrium State, E^0

At the cultist-free equilibrium, there is absence of cultists. Thus, all the infected classes will be zero.

Lemma 3.1: A cultist-free equilibrium state of the model exists at the point

$$\left.\begin{aligned} S &= \frac{\wedge\theta}{\mu} \\ S_c &= \frac{\wedge(1-\theta)}{\mu} \\ C &= 0 \\ A &= 0 \\ E &= 0 \\ G &= \frac{a}{b} \\ N_1 &= \frac{\wedge\theta}{\mu} \\ N_2 &= \frac{\wedge(1-\theta)}{\mu} \\ N &= \frac{\wedge}{\mu} \end{aligned}\right\} . \tag{16}$$

By simplification and some substitutions in (2), we have

$$\lambda^* = 0 \tag{17}$$

or

$$[(\mu+\lambda)(\mu b+\delta_3 a)-\delta_3 a\lambda]-\beta b\mu = 0. \tag{18}$$

3.4. Local Stability of Cultist-free Equilibrium and Basic Reproduction Number, R_0

The stability of cultist-free equilibrium was established using a next-generation matrix. Let $F_i(x)$ be the rate of appearance of new infections in the infected compartment and $V_i(x)$ be the next decreasing rate of infections in the compartment due to infected flow in the system of infected compartments. The next generation matrix is the method used to derive R_0, for a compartmental model of spread of cultism. If $F=(f_1,---,f_2n)$ and $V=(v_1,---,v_2n)$, let partition the derivative $\Delta F(E_0)$ and $\Delta V(E_0)$ as

$$F(x_0)=\begin{bmatrix}\frac{dF_1}{dS_c^0} & \frac{dF_1}{dC^0}\\ \frac{dF_2}{dS_c^0} & \frac{dF_2}{dC^0}\end{bmatrix} \qquad V(x_0)=\begin{bmatrix}\frac{dV_1}{dS_c^0} & \frac{dV_1}{dC^0}\\ \frac{dV_2}{dS_c^0} & \frac{dV_2}{dC^0}\end{bmatrix}$$

where

$$F=\begin{bmatrix}0 & -\beta\\ 0 & \beta\end{bmatrix}, \tag{19}$$

$$V=\begin{bmatrix}\mu & 0\\ 0 & \frac{\mu b+\delta_3 a}{b}\end{bmatrix} \tag{20}$$

$$and \qquad V^{-1}=\begin{bmatrix}\frac{1}{\mu} & 0\\ 0 & \frac{b}{\delta_3 a+\mu b}\end{bmatrix}. \tag{21}$$

Then, multiplying (19) and (21), it becomes

$$FV^{-1}=\begin{bmatrix}0 & \frac{-\beta b}{\delta_3 a+\mu b}\\ 0 & \frac{\beta b}{\delta_3 a+\mu b}\end{bmatrix} \tag{22}$$

Hence, the reproduction number is obtained as $R_0=\frac{\beta b}{\delta_3 a+\mu b}$.
Therefore, the reproduction number is denoted by $R_0=\rho FV^{-1}$, where ρ denotes the spectral radius. Thus, We established that the cultist-free equilibrium of the model is locally asymptotically stable if $R_0<1$, that is cultist dies out and unstable if and only if $R_0>1$ meaning that the cultism activities will persist
The basic reproductive number, R_0 is the key measure in estimating the ability of cultism to spread. It is the average number of secondary transmissions from one infected person; when R_0 is greater than 1, the epidemic is growing.

3.5. Existence of Local Stability of Cultist-Free Equilibrium

The Routh-Hurwitz criteria are employed to ascertain the asymptotic stability of an equilibrium in a non-linear system of differential equations. These criteria offer both necessary and sufficient conditions, ensuring that all roots of the characteristic polynomial include negative components, thereby guaranteeing asymptotic stability (Markin, 1997). The local stability of the equilibrium can be deduced from the Jacobian matrix given as follows.

$$J(E^0)=\begin{bmatrix}-\mu & 0 & -m_1 & 0 & 0 & 0\\ 0 & -\mu & -m_2 & 0 & 0 & 0\\ 0 & 0 & m_4 & 0 & 0 & 0\\ 0 & 0 & m_5 & -m_6 & 0 & 0\\ 0 & 0 & 0 & m_7 & -\mu & 0\\ 0 & 0 & 0 & 0 & 0 & -b\end{bmatrix} \tag{23}$$

where

$$\begin{aligned} m_1 &= \frac{\kappa\wedge\theta}{\mu}\\ m_2 &= \beta\\ m_4 &= \frac{\beta b-\delta_3 a-\mu b}{b}\\ m_5 &= \frac{\kappa\wedge\theta}{\mu}\\ m_6 &= \frac{(\tau_2+\delta_2)a+(\mu+\tau_1+\delta_1)b}{b}\\ m_7 &= \frac{\tau_1 b+\tau_2 a}{b}. \end{aligned}$$

Therefore, the cultist-free equilibrium is locally asymptotically stable when there is no cultist. It follows that the characteristic equation (23) by using $|J(E^0-\lambda I)|$ computed is given by solving the determinant with maple software which yields to

$$(-\mu-\lambda)^3(m_4-\lambda)(-m_6-\lambda)(-b-\lambda). \tag{24}$$

Simplifying (24), gives

$$\begin{aligned} &\lambda^6+(3\mu-m_4+m_6+b)\lambda^5+(3\mu^2-3\mu m_4\\ &+(3\mu-m_4)m_6-(-3\mu+m_4-m_6)b)\lambda^4\\ &+(\mu^3-3\mu^2m_4+(3\mu^2-3\mu m_4)m_6-(-3\mu^2\\ &+3\mu m_4-(3\mu-m_4)m_6)b)\lambda^3+(-\mu^3m_4\\ &+(\mu^3-3\mu^2m_4)m_6-(-\mu^3+3\mu^2m_4-(3\mu^2\\ &-3\mu m_4)m_6)b)\lambda^2+(-\mu^3m_4m_6-(\mu m_4\\ &-(\mu^3-3\mu^2m_4)m_6)b)\lambda-\mu^3m_4m_6b \end{aligned}$$

Collecting the coefficient of the eigenvalues, λ and characteristic roots gives

$$\lambda^6+x_5\lambda^5+x_4\lambda^4+x_3\lambda^3+x_2\lambda^2+x_1\lambda+x_0=0, \tag{25}$$

where

$$x_6 = 1$$
$$x_5 = (3\mu - m_4 + m_6 + b)$$
$$x_4 = (3\mu^2 - 3\mu m_4 + (3\mu - m_4)m_6 - (-3\mu + m_4 - m_6)b)$$
$$x_3 = (\mu^3 - 3\mu^2 m_4 + (3\mu^2 - 3\mu m_4)m_6 - (-3\mu^2 + 3\mu m_4 - (3\mu - m_4)m_6)b)$$
$$x_2 = (-\mu^3 m_4 + (\mu^3 - 3\mu^2 m_4)m_6 - (-\mu^3 + 3\mu^2 m_4 - (3\mu^2 - 3\mu m_4)m_6)b)$$
$$x_1 = (-\mu^3 m_4 m_6 - (\mu m_4 - (\mu^3 - 3\mu^2 m_4)m_6)b)$$
$$x_0 = -\mu^3 m_4 m_6 b.$$

Using the Routh-Hurwitz criterion it can be seen that all the eigenvalues have negative real parts and therefore, the cultist-free equilibrium is locally asymptotically stable.

3.6. Global Stability of Cultist-Free Equilibrium, (E^0)

Equilibrium's global stability eliminates constraints on model variable initial conditions. In the case of global asymptotic stability, solutions converge to equilibrium regardless of initial conditions. The proof of global stability for the disease-free equilibrium often involves methods like the Lyapunov theorem, which was employed in this study. Volterra, in 1920, developed the Lyapunov function in population dynamics, offering a sufficient condition for global asymptotic stability at equilibrium.

Theorem 3.3: The cultist-free equilibrium, E^0 of the model is globally asymptotically stable in Ω if $R_0 < 1$.
Proof: Consider the Lyapunov function

$$M = pC + qE, \tag{26}$$

Where p and q are constant to be determined.
By differentiating equation (26) we have

$$M' = pC' + qE'. \tag{27}$$

By substituting and simplifying (27), gives

$$M' = p\lambda S_c - pC(\mu + \delta_3 G) - qA(\tau_1 + \tau_2 G) + q\mu E \tag{28}$$

Setting the coefficient of λS_c equal to the numerator of R_0 ignoring β., then

$$p = -b. \tag{29}$$

Setting the coefficient of pC from (28) equal to the denominator of R_0 implies that

$$b = -1. \tag{30}$$

by substituting (30) in (29), we have

$$p = 1 \tag{31}$$

Set the coefficient of E from (28) equal to zero we have

$$q = 0. \tag{32}$$

by substituting (31) and (32) in (27)

$$M' = C'. \tag{33}$$

Substituting C' in (33) and putting the coefficient of pC as denominator, we have

$$M' = \frac{\lambda S_c - (\mu + \delta_3 G)C}{\mu + \delta_3 G} \tag{34}$$

which implies that

$$and \qquad M' = \frac{\frac{\beta C S_c}{N_2} - (\mu + \delta_3 G)C}{\mu + \delta_3 G}, \tag{35}$$

i.e

$$M' = (\mu + \delta_3 G)C\left\{\frac{\beta}{\mu b + \delta_3 G} - 1\right\}. \tag{36}$$

Since $N_2 \le N_2^0, S_c \le S_c^0, G \le G^0$,
Then

$$M' \le (\mu + \delta_3 G^0)C^0\left\{\frac{\beta}{\mu b + \delta_3 G^0} - 1\right\}, \tag{37}$$

i.e

$$M' \le \frac{1}{b}(\mu b + \delta_3 a)C^0\left\{R_0 - 1\right\}. \tag{38}$$

Since the model parameters are non-negative, it follows that $M \le 0$ for $R_0 \le 1$, with $M = 0$ if and only if $C = F = 0$. Hence, M is the Lyapunov function in the invariant region. Thus the above theorem shows that the sociological requirement of $R_0 \le 1$ is a sufficient condition for the elimination of cultism.

4. Existence of Endemic Equilibrium State $(\mathbf{E}^{**})$

The endemic equilibrium state or point is a positive steady state solution at which the cultist persists in the population. Therefore, the model equations (1) has an endemic equilibrium point, E^{**}. The endemic equilibrium of equations (1) is locally asymptotically stable. However, when $R_0 > 1$, the cultist recruitment will remain at high.

Lemma 3.2: The endemic equilibrium state of the model (1) exists if the basic reproduction number

$R_0 > 1$.

Proof: At the endemic equilibrium state, let

$$\left.\begin{aligned} S^{**} &= S \\ S_c^{**} &= S_c \\ C^{**} &= C \\ A^{**} &= A \\ E^{**} &= E \\ G^{**} &= G \end{aligned}\right\} \tag{39}$$

Consider an arbitrary equilibrium at which equation (18) holds, i.e

$$[(\mu + \lambda)(\mu b + \delta_3 a) - \delta_3 a\lambda] - \beta b\mu = 0.$$

By simplification, we get

$$\mu^2 b + \mu\delta_3 a - \beta b\mu + \mu b\lambda = 0. \tag{40}$$

let

$$X_1 = \mu b\lambda \tag{41}$$

$$X_2 = \beta b\mu - \mu^2 b - \mu\delta_3 a \tag{42}$$

Thus (18) becomes

$$X_1\lambda + X_2 = 0 \tag{43}$$

Using Descartes's rule of sign, there exits a unique positive equilibrium if

$$X_2 > 0 \tag{44}$$

$$\beta b\mu - \mu^2 b - \mu\delta_3 a > 0 \tag{45}$$

$$\beta b\mu \geq \mu(\mu b + \delta_3 a) \tag{46}$$

$$\frac{\beta b}{(\mu b + \delta_3 a)} > 1 \tag{47}$$

$$R_0 > 1. \tag{48}$$

Thus, $\lambda > 0$ if $R_0 > 1$. Then substituting $\lambda > 0$ at E^{**} gives

$$\left.\begin{aligned} S^{**} &> 0 \\ S_c^{**} &> 0 \\ C^{**} &> 0 \\ A^{**} &> 0 \\ E^{**} &> 0 \\ G^{**} &> 0 \end{aligned}\right\} \tag{49}$$

Hence, the endemic equilibrium state of the model exits if $R_0 > 1$.

4.1. Local Stability of Endemic Equilibrium

The Routh-Hurwitz criteria is used to establish asymptotic stability of equilibrium for a non-linear system of differential equations. The local stability of the equilibrium may be determined from the Jacobian matrix,

using elementary row operation yields to

$$J(E^0) = \begin{bmatrix} -m_1 & 0 & -m_2 & 0 & 0 & 0 \\ 0 & -m_3 & -m_4 & 0 & 0 & 0 \\ 0 & 0 & -m_6 & 0 & 0 & m_7 \\ 0 & 0 & 0 & -m_{10} & 0 & m_{11} \\ 0 & 0 & 0 & 0 & -m_{13} & m_{14} \\ 0 & 0 & 0 & 0 & 0 & -m_{15} \end{bmatrix} \tag{50}$$

where

$$\begin{aligned} m_1 &= (\kappa C + \mu) \\ m_2 &= \kappa S \\ m_3 &= (\frac{\beta C}{N_2} + \mu) \\ m_4 &= \frac{\beta S_c}{N_2} \\ m_5 &= \frac{\beta C}{N_2} \\ m_6 &= (-\frac{\beta S_c}{N_2} + (\mu + \delta_3 G)) \\ m_7 &= \delta_3 C \\ m_8 &= \kappa C \\ m_9 &= \kappa C \\ m_{10} &= ((\tau_2 + \delta_2)G + (\mu + \tau_1 + \delta_1)) \\ m_{11} &= (\tau_2 + \delta_2)A \\ m_{12} &= (\tau_1 + \tau_2 G) \\ m_{13} &= \mu \\ m_{14} &= (\tau_1 + \tau_2)A \\ m_{15} &= b. \end{aligned}$$

Theorem 3.4: Endemic equilibrium is locally stable if $R_0 > 1$.

Proof:
let

$$
\begin{aligned}
\lambda_1 &= -(\kappa C + \mu) < 0 \\
\lambda_2 &= -(\frac{\beta C}{N_2} + \mu) < 0 \\
\lambda_3 &= -(-\frac{\beta S_c}{N_2} + (\mu + \delta_3 G)) < 0 \\
\lambda_4 &= -((\tau_2 + \delta_2)G + (\mu + \tau_1 + \delta_1)) < 0 \\
\lambda_5 &= -\mu < 0 \\
\lambda_6 &= -b < 0.
\end{aligned}
$$

The positive endemic equilibrium state of the system (1) is locally asymptotically stable when $R_0 > 1$. Ninuola (2017).
The sociological implication of this theorem is that cultism will continue to persist in the population when $R_0 > 1$ and the initial size of the sub-populations of the model are in the basin of attraction of the endemic state.

5. NUMERICAL VERIFICATION

In this section, the simulations were carried out to verify and validate the analytical results on the cultist-free equilibrium and endemic equilibrium of the model using the various variables and parameters for initial conditions.

5.1. Parameter Estimation

Table 1.2: Values of parameters of the model

S/N	Parameters	Values	Source
1	$\wedge$	20000000	Hypothetical
2	θ	0.9	Hypothetical
3	κ	0.0002	Hypothetical
4	δ_1	0.1	Hypothetical
5	δ_2	0.17	Hypothetical
6	τ_1	0.9	Hypothetical
7	β	0.9	Hypothetical
8	a	0.0085	Calculated
9	b	0.01	Calculated
10	μ	0.017	Calculated
11	τ_2, δ_3	(0-1)	Controls

The recruitment rate, $\wedge$ in the susceptible sub-population is assumed to be 20000000 while the average lifespan of individual is $\frac{1}{\mu}$ which gives a death rate, μ of 0.017. Government capacity is calculated to be 0.0085 and government relaxation is calculated to be 0.01. Other values of the model are hypothetical.

5.2. Numerical Simulations

The simulations were carried out using the following parameters from Table 1.2 for initial conditions. The final time was t = 15 months. Computations were run in maple17 software for the analysis.

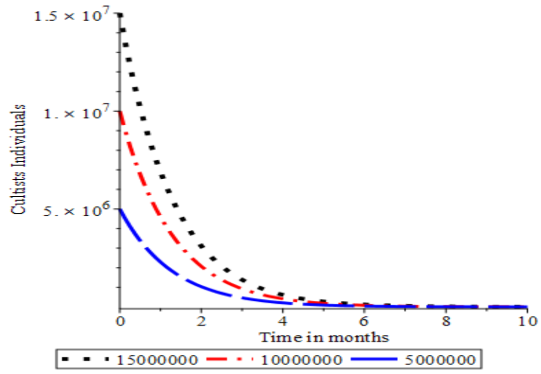

Figure 2: The total number of Cultists with different initial variables: $C_1 = 15000000$**, **$C_2 = 10000000$**, and** $C_3 = 5000000$**, with control parameters from Table 1.2:** $\beta = 0.9, \tau_2 = 0.017, \delta_3 = 0.0117, \mu = 0.017$, **which gives** $R_0 = 33.40$

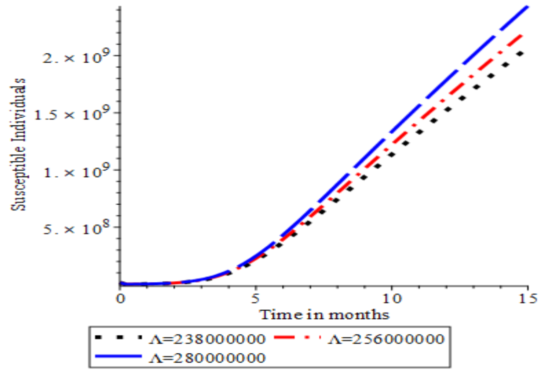

Figure 3: The simulation of Susceptible with $S = 15000000$**, and different recruitment rates:** $\wedge_1 = 238000000, \wedge_2 = 256000000$, **and** $\wedge_3 = 280000000$ **with control parameters from Table 1.2:** $\kappa = 0.0002, \theta = 0.9, \tau_2 = \delta_3 = \mu = 0.017$**. This gives** $R_0 = 28.62$

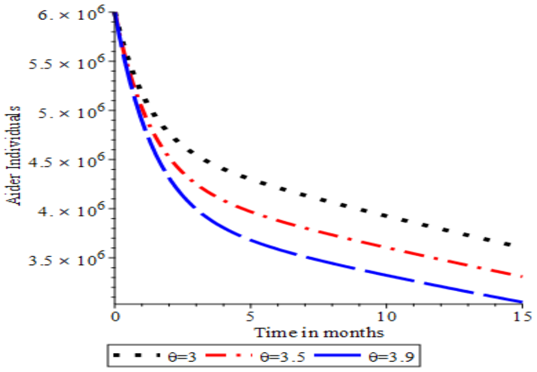

Figure 4: The graph of Aiders with different proportion rates of non-recruitable individuals at $\theta_1 = 3, \theta_2 = 3.5$, and $\theta_3 = 3.9$, with control parameters from Table 1.2: $\kappa = 0.0002, \delta_1 = 0.1, \delta_2 = \delta_3 = \mu = 0.017, \tau_1 = 0.9, \tau_2 = 0.1$.

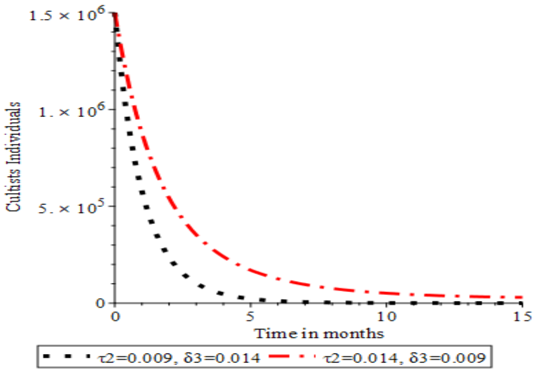

Figure 5: Graph of cultist with initial $C = 15000000$ **and two controls compared:** $\tau_2 = 0.009, \delta_3 = 0.014$ **and** $\tau_2 = 0.014, \delta_3 = 0.009$

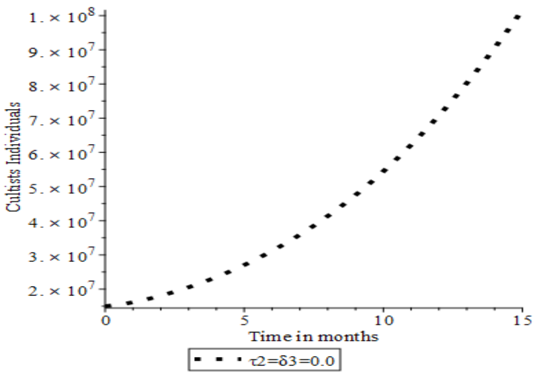

Figure 6: Graph of cultist with initial $C = 15000000$ **and without controls :** $\tau_2 = 0, \delta_3 = 0$

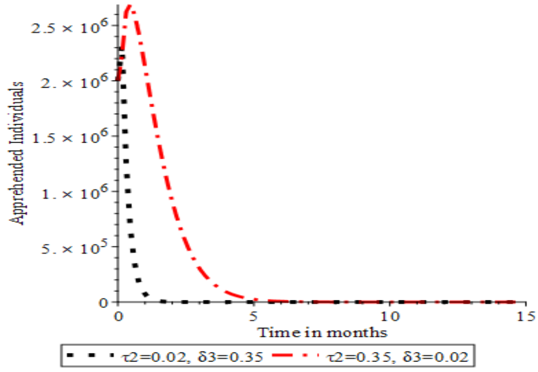

Figure 7: Simulation of apprehended compare with two different controls: $\tau_2 = 0.02, \delta_3 = 0.35$ **and** $\tau_2 = 0.35, \delta_3 = 0.02$

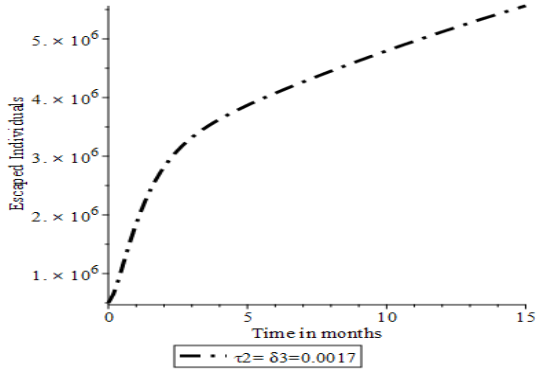

Figure 8: The simulation of escaped with controls: $\tau_1 = \mu = 0.017, \tau_2 = \delta_3 = 0.0017$

5.3. Discussion of Results

Figure 2 shows that with government efforts, the system converges at cultist-free equilibrium in all different initial variables which entails that with government control, the cultists will be eliminated. Hence, showing the global asymptotic stability of the endemic equilibrium. Hence, with $R_0 > 1$, it clearly shows that the global asymptotic stability of the endemic equilibrium and the persistence of the system profiles. Also in Figure 3, it was observed that the system profiles converge to the cultist-free equilibrium in all cases of recruitment with the effects of government efforts. Hence, the analytical result is globally asymptotically stable of cultist-free equilibrium. However, The simulation in Figure 4 shows that the higher the number of susceptible, the easier for the government to control the population. The solution profile results are locally asymptotically stable of the endemic equilibrium of the cultist-free equilibrium. In Figure 5, eliminating cultists is the best way of controlling cultism. By observation, this figure is locally and globally asymptotically stable. While Figure 6 shows that without government efforts, the

system of cultists will exponentially increase. This reveals that the simulation is locally asymptotically stable of the endemic equilibrium.
Figure 7 shows that the elimination of cultists is an efficient way of reducing this population, followed by the government rescuing the apprehended individuals. This system shows the global asymptotic stability of the endemic equilibrium. Figure 8 indicates that without control, several susceptible individuals have been captured by cult members and the population of apprehended individuals has increased to the peak.

6. Conclusion

This work demonstrated the targeted control strategies such as killing and taking cultist to jail can effectively reduce cultism incidents. The results of this work might be extended to other fields of study to investigate the prevalence of other social menace such as kidnapping, banditry, violence, etc. who seriously endanger public safety in Nigeria.
The result of the numerical computation carried out indicates that eliminating cultists by the government reduces the cults in the population. Hence, an effective way of controlling cultism in society.

7. Recommendations

Considering the efforts of the government in reducing cultism, here are recommendations based on the study:

1. **Strengthening Anti-Cultism Laws:** Acknowledge the government's introduction of stringent Anti-cultism Laws and recommend continuous efforts to strengthen and enforce these laws. Emphasize the importance of legal measures in curbing cult-related activities.

2. **Enhancing Law Enforcement Units:** Highlight the establishment of specialized law enforcement units, such as the Anti-cultism Unit of the Nigerian Police Force. Suggest ongoing support and resources for these units to effectively dismantle cults and maintain a safe academic environment.

3. **Community Engagement and Awareness:** Advocate for community engagement and awareness campaigns to educate students, parents, and communities about the dangers and consequences of cultism. This could involve partnerships with educational institutions, NGOs, and community leaders to promote a collective effort against cult activities.

4. **Promoting Alternative Youth Empowerment:** Recognize the socio-economic challenges in Nigeria during the 1970s, 1980s, and 1990s that contributed to the proliferation of cults. Recommend government initiatives focused on providing alternative avenues for youth empowerment, addressing economic challenges, and fostering a sense of belonging without resorting to cultism.

5. **Collaboration with Educational Institutions:** Encourage collaboration between the government and educational institutions to implement proactive measures within campuses. This may include establishing counseling services, mentorship programs, andcreating a supportive environment to deter students from engaging in cult activities.

6. **Research and Continuous Monitoring:** Support ongoing research efforts, including mathematical modeling, to understand and predict the dynamics of cultism. Advocate for continuous monitoring and evaluation of the effectiveness of government interventions and adjust strategies based on research findings.

7. **International Collaboration:** Suggest international collaboration on best practices for tackling cult-related issues. Exchange information and experiences with other countries facing similar challenges to adopt effective strategies in the Nigerian context.

By incorporating government efforts, the government can further enhance its capabilities to reduce cultism and promote a safer and conducive learning environment in Nigerian higher institutions.
This research can further be studied and expanded to other fields of knowledge by adding or reducing variables and parameters to the model.

Conflict of interest

On behalf of all authors, I assure you that there is no conflict of interest.

Authors Contributions

All the Authors contributed in one way or the other

Ethical Conduct

The authors wrote this article with good faith and respect consistent with laws, regulations, and policies governing our conduct.

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REFERENCES

Adamu, G. & Ibrahim, M.O.(2020). Mathematical modeling of dynamics behavior of terrorism and control. *Caspian Journal of Mathematical Sciences* (CJMS) University of Mazandaram, Iran. 9(1), 68-89

Amunnadi C.A. (2017) Addressing the high spate of negative attitudes and misconducts in Nigerian tertiary education system through ethical re-engineering *Online Journal of Arts, Management and Social Sciences.* (OJASS). Vol.2,3, 53-63.ISSN:2276-9013 ISSN:

Ajayi, (2010). Menace of Cultism in Nigerian Tertiary Institutions: The Way Out. *Anthropologist*, 12(3): 155-160

Akanni J.O & Abidemi A.(2021). Mathematical Analysis of the Concomitance of Illicit drug use and banditry in population. *Journal of Scientific and Engineering Research*, 5(8).Doi:10.21203/rs-1089919/v1.

AkanniJ.O.(2020). Asymptotic Stability Of Illicit Drug Dynamics with Banditry Compartmen.*Applied Mathematics & Information Sciences International Journal.* 14(5): 791-800.

Aniayam, B., Okrinya, J.& Consul, I. (2018). A Mathematical Modelling of Kidnapping with rehabilitation.. *Journal of Scientific and Engineering Research*, 5(8), 195-205.

Benson, (2022). Risk Assessment and the Impact of Cultism Crisis in the Nigerian Communities. *Archives of Current Research International* 22(3):25-40.

Brauer (2012). Agent-Based Modeling of Cultism: A pervasive issue in societies worldwide.

Daniel M.I.(2022). Banditry and Security Threats: An Analysis of Insecurity in Nigeria. *Journal of Public Administration, Finance and Law* . Issue 23. https://doi.org/10.47743/jopafl-2022-23-04

Hussain S. (2019). Dynamical behavior of a mathematical model on the network of militants. *Punjab University Journal of Mathematics* , 51(1), 51-60.

John O.A & Afeez A.(2023). Relationship between Illicit Drug Users and Bandits in a Population: Mathematical Modelling Approach. *Applied Mathematics & Information Sciences International Journal* Volume 7, Issue 3, pp.475-492.

Sunday O., Samuel O. & Folahanmi A.(2023). Forces of Terror: Armed Banditry and Insecurity in North-west Nigera. *Democracy and Security¿* DOI: 10.1080/17419166.2023.2164924.

Jibril L. & Amiru S.(2023). Modeling and Optimal Control Analysis on Armed Banditry and Internal Security in Zamfara State.*European Journal of Theoretical and Applied Sciences.* 1(5), 1062-1075. DOI: 10.59324/ejtas.2023.1(5).93

Ninuola (2017). Stability Analysis of Endemic Equilibrium Points of Malaria and Dengue Fever Coinfection Model. *International Journal of Scientific Research in Mathematical and Statistical Sciences* Volume 7, Issue 6, pp.34-45.

Ogunade, R. (2002). Secret Societies and Cultist Activities in Nigeria Tertiary institutions in leading issues in General Studies. *Ilorin, University pf Ilorin press.*

Okoye, C., Collins, O.C., & Mbah, G.C.E. (2020). A mathematical approach to the analysis of terrorism dynamics. *Security journal*

Okwu O.J. (2006). A critique of students' vices and the effect on the quality of graduates of Nigerian tertiary institutions. *Journal of Social Sciences*, 12(3): 193-198.

Osakinle E.O. & Falana B.A. (2011).

Salisu (2021). Mathematical Model for the Dynamics of Drug-Abuse and Cultist Violence Co-Menace. *The Pacific Journal of Science and Technology.* Volume 22, pp. 47-58.

Tasiu A.R., Shamsu S.M., S. Abdullahi and Junaidu H.(2024) Mathematical modelling of kidnapping: dynamics and control.*Umyu Scientifica* Vol.3,105-117 Doi.org/10.56919/usci2433.013

Udoh, (2015). Causes, Effects, and Strategies for Eradicating Cultism among Students in Tertiary Institutions in Nigeria. *Journal of Education and Practice* . Vol.6, No.22.

Thomas, D. (2002). F9 Cultist. 15(18): pp 12-14.